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ATTENUATION OF LOW FREQUENCIES FOR DUCTED SYSTEMS BASED ON THE ACOUSTIC BLACK HOLE TECHNIQUE

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The Acoustic Black Hole (ABH) is a new technique that was proposed for passive vibration control but has been recently extended for broadband noise control in ducted geometries. For instance, in areas such as building and transportation industries, the control of low-frequency noise propagating in exhaust or air-conditioning waveguides constitutes an important issue and a very challenging problem. Although many studies have presented closed ABH geometries, the used of open-ended ABHs is compulsory in duct systems where a mean axial flow is present. In this case, the concept of sound trapping involves minimization of both sound reflection and transmission inside the retarding structures. This work is focused on the optimal determination of the ABH physical constituting parameters that mostly influence the metamuffler performance. The requirements on minimal volume impose a constraint for the total dissipation that has to be considered regarding low-frequency broadband operation and manufacturing constraints. An analytical formulation will be presented in terms of the Transfer Matrix Method to model a widely-opened silencer composed of a finite number of sidebranch cavities with an axial variation of their depth according to an exponential law. These cavities are separated by rigid rings of constant thickness. Assuming plane wave propagation along the main duct and within the annular cavities, visco-thermal effects are considered to calculate the power dissipated by the ABH. Influence of several parameters is studied considering the optimal axial rate of cavity depth variation, the wall porosity and the ratio between the total silencer length and the duct radius. The simulated optimal results are verified against a set of measurements performed in a standing wave tube where the ABH is inserted. A good agreement is found between both values that validates the proposed metamuffler with near-unit dissipation over its efficiency broad bandwidth.

Keywords: acoustic black hole, duct acoustics, sound dissipation, sound retarding structures.

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1. Introduction

Cancellation of low frequency noise where traditional passive techniques with limited size are not effective is an important technological issue. For instance, attenuation of sound in rotating duct systems cannot be effectively handle by classical sound absorbing materials [1] due to the low-broadband frequency content present in the spectrum. Micro-perforated [2] mufflers have been shown to be able to extend the performance towards the low frequency range provided that an optimal combination of several parameters are able to constitute a multi-layer partition [3] with typical dimensions related to the lowest frequency of control.

Recently, a new type of solutions has appeared for perfect absorption of sound based on artificial engineering materials [4]. Metamaterials properties are based on the interaction between structural building blocks and precise control of the internal microstructures embedded in a background medium that can provide key effective parameters that are not constrained by the properties of their constituents. Sound-wave propagation is controlled by the effective mass density ρ and bulk modulus κ of a material. These properties can be selected for extreme manipulation of sound. In particular, attenuation of low-frequency noise has been studied considering structures that can absorb almost 100% of the incident energy with reduced dimensions [5]. A review of different metamaterials based on local resonances that achieve broadband absorption spectra within the subwavelength-scaled structures. They have also provided a complete analysis on the design of broadband absorbers with a minimum length according to the principle of causality [6].

Acoustic waveguides have attracted great attention for practical implementation of acoustic metamaterials. One of the earliest examples can be found using a rigid-walled duct with a periodic arrangement of side-branch cavities. The introduction of periodic resonant inclusions induce negative effective properties that create ‘stop-bands’ where sound transmission is forbidden. This has been verified using the Bloch formalism for a set of periodic Helmholtz resonators branched to a uniform flow duct through micro-perforated necks [7]. A new concept, denoted as Acoustic Black Hole (ABH) has been proposed based on the principle of "slow-sound" generation that seems promising for energy trapping. These control devices have appeared initially in the field of vibration control, using thin-walled structures, such as beams or plates. The profile at the edge presents a decaying power-law fitted for a gradual reduction of the vibrational wave along the structure boundary thickness [8]. Recently, Deng *et al.* [9] proposed a composite structure with periodic additive ABHs to absorb vibration from plates.

The same concept has been extrapolated for the control of airborne sound in waveguides geometries. They are retarding structures able to produce a progressive reduction of the propagating wave velocity. These devices can also be interpreted as functionally-graded materials with a structure geometry that changes in a continuous or stepwise way along the axial position. They are conceived with the idea of fully dissipating the incident excitation within the system. This effect is achieved due the phenomenon of slow-sound propagation along the direction of the duct channel and visco-thermal dissipation in the sidebranch cavities. The cavities become then sub-wavelength trapping resonators. A first practical realization of ABHs was presented by Mironov [10] using a one-dimensional axially wall impedance that was adjusted using a set of rigid discs with parabolic increase of their diameters and decrease of their axial interspacing. Initial cases were dealing with totally or partially-closed ABHs were the goal was to reduce reflected waves [11]. The Transfer Matrix Method (TMM) has been proposed as an analytical tool to examine different duct terminations [12, 13].

The use of widely-opened ABHs has become a subject of interest for simultaneous reduction of both reflection and transmission [14] and the understanding of the physical mechanism involved for the slow-sound effect and the critical coupling condition [15, 16]. Considering this geometry, some authors have started to study the presence of a flow along the axial dimension [17].The general objective of the present work is the study of the ABH effect to design graded metamaterials and achieve ultra-slow sound conditions and low-frequency wideband attenuation performance for applications in ducted flows. Properly selection of the parameters has proved to be essential for an optimal performance of the metamuffler.

For instance, study of the cavity profile defined by a power law variation has been optimized using evolutionary strategies with covariance matrix adaptation [18], or using a topology optimization approach [19]. The performance of the ABH can be improved by filling the bottom of the cavities with absorbing material, with a distribution highly dependent on the optimal profile [18]. In this work we will use the analytical TMM for simulating the ABH broadband properties and optimizing its main parameters for enhanced performance in the low frequency range. The predictions are validated with measurements in a transmission impedance tube. Conclusions and future lines will be outlined at the end.

2. Analytical formulation

2.1 Wave propagation modelling

The system that will be considered is presented in Figure 1. It is a fully-opened metamuffler with inner radius R and total length L, that is limited to a maximum value of 0.15 m as a constraint for a compact system.

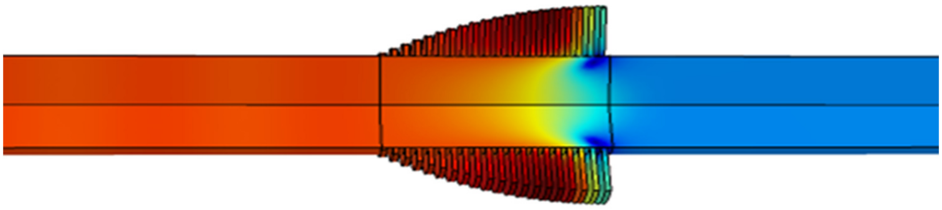

Figure 1: Schematic representation of a metamuffler based on the slow-sound effect.

The ABH is composed of a set of outer cavities with increasing depth that extends between the inlet at $z=-L$ towards the outlet at $z=0$, as indicated in Figure 2. Considering the frequency convention $\mathrm{e}^{\mathrm{j}\omega t}$, the linearized mass conservation equation for a duct wall impedance varying along the axis is provided by [12]

$$\frac{\partial v_z}{\partial z}+\frac{v_n}{r_H}+\frac{\mathrm{d}(\log S)}{\mathrm{d}z}v_z=-\frac{\mathrm{j}\omega}{\rho_0 c_0^2}p, \quad -L<z<0, \tag{1}$$

with v_z the axial velocity component, $v_n=p/Z$ the normal velocity over the boundary $\Sigma(r=R,-L<z<0)$ and p the acoustic pressure. $r_H=S/U$ is the hydraulic radius with S the cross-sectional area of the duct and U the circumference of the lining. The wall admittance of the ABH constituted of a continuous distribution of annular cavities can be approximated in the low frequency range $(k_0R<<1)$ by

$$y(z)=\mathrm{j}k_0\left[R^2-(R+D(z))^2\right], \tag{2}$$

with $k_0=\omega/c_0$ and $D(z)$ the cavity depth increasing along the axis. When considering the linearized momentum conservation equation, one obtains a plane wave equation of the form [20]

$$\frac{\mathrm{d}^2p}{\mathrm{d}z^2}+k_0^2\left[1+\frac{2}{R}\left(D(z)+\frac{D^2(z)}{2R}\right)\right]p=0, \tag{3}$$

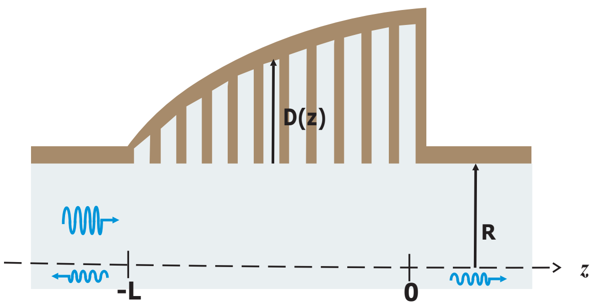

Figure 2: The fully-opened ABH configuration with the main geometrical parameters.

that corresponds to a Helmholtz-type equation where the wall cavity profile increases according to the law $D(z)=R(1-\varphi_m(z))$, with $\varphi_m(z)=(-z)^m/L^m$, $m>0$. Considering this expression, the phase speed of the acoustic wave propagating downstream along the axis can be expressed as [20]

$$c_z(z)=\frac{c_0}{2-\varphi_m(z)}. \tag{4}$$

It can be outlined that the acoustic wave phase speed decreases progressively from c_0 at the inlet towards $c_0/2$ at the ABH outlet. This situation is clearly different from the closed ABHs where the phase and group velocity decrease to zero when approaching the ABH termination [13].

2.2 Analytical formulation

The Transfer Matrix Method (TMM) has been used for the analysis of the widely-opened muffler considering that this device is composed of a set of ring sections separated by air cavities and distributed over an overall length L along the axial dimension. The radii of the air cavities in the fully-opened silencer progressively increase from 0 to R following an expanding power law from the inlet situated at $z=-L$ towards the outlet at $z=0$. Assuming plane wave propagation, the local side-branch volume admittance at $z=z_i$ is given by $Y_{\mathrm{cav},i}=S_{\mathrm{cav}}\,y(z_i)/Z_0$, with $y(z_i)$ given by Eq. (2), $S_{\mathrm{cav}}=2\pi Rd$ the cavities entrance area and d their width. Applying continuity of the acoustic pressure and acoustic flow rate across the i^{th} cavity-ring unit leads to the relationship, $[p_i\ u_i]^{\mathrm{T}}=\mathbf{T}_i[\mathrm{p}_{i+1}\ u_{i+1}]^{\mathrm{T}}$, between the pressure and volume velocity fields at the input interface $[p_i\ u_i]^{\mathrm{T}}$ and those at the output interface $[p_{i+1}\ u_{i+1}]^{\mathrm{T}}$, with $\mathbf{T}_i$ the associated transfer matrix given by

$$\mathbf{T}_i=\begin{bmatrix}\cos(k_0 d) & \mathrm{j}\dfrac{Z_0}{S}\sin(k_0 d)\\ \mathrm{j}\dfrac{S}{Z_0}\sin(k_0 d) & \cos(k_0 d)\end{bmatrix}\cdot\begin{bmatrix}\cos(k_0 d_t) & \mathrm{j}\dfrac{Z_0}{S}\sin(k_0 d_t)\\ \mathrm{j}\dfrac{S}{Z_0}\sin(k_0 d_t) & \cos(k_0 d_t)\end{bmatrix}\begin{bmatrix}1 & 0\\ Y_{\mathrm{cav},i} & 1\end{bmatrix} \tag{5}$$

The overall transfer matrix $\mathbf{T}$ between the inlet and the outlet satisfies $[p_1\ u_1]^{\mathrm{T}}=\mathbf{T}[p_{N+1}\ u_{N+1}]^{\mathrm{T}}$ and is expressed as the product of the transfer matrices, $\mathbf{T}=\prod_{i=1}^{N}\mathbf{T}_i=\begin{bmatrix}T_{11} & T_{12}\\ T_{21} & T_{22}\end{bmatrix}$. The expressions for the reflection and transmission coefficients are given by

$$\begin{cases} r = \dfrac{(T_{11} + z_0 T_{12}) - (T_{21} + z_0 T_{22})}{(T_{11} + z_0 T_{12}) + (T_{21} + z_0 T_{22})} \\ t = \dfrac{1+r}{T_{11} + z_0 T_{12}} \end{cases} . \tag{6}$$

with $z_0 = S/Z_0$. The JCAL model of visco-thermal losses inside the cavities has been used [20]. The power dissipated by the ABH then reads $\eta = 1 - |r|^2 - |t|^2 = \alpha - \tau$ with α the absorption coefficient and τ the transmission coefficient. The transmission loss (TL) is defined as $\mathrm{TL(dB)} = -10\log_{10}(\tau)$.

3. Parametric study

The analytical model outlined in the previous section has been used to perform a parametric study on the physical and geometrical characteristics that can have an influence on the ABH final performance. Due to the large number of different physical parameters that can be selected, the selection of the optimal values for best performance is a high-cost computational problem that cannot be solved with traditional optimization methods. Here we will present a parametric study to show the variation of the results on two different quantities. The nominal muffler presents a radius $R = 0.047$ m and length $L = 0.1$ m, with a set of $N = 10$ annular cavities of axial width $d = 0.008$ m separated by ring walls of thickness $d_t = 0.002$m, thus leading to a wall porosity $\sigma = d/(d + d_t) = 80\,\%$. Fig. 3 presents the simulated results for the dissipation, reflection and transmission loss power spectra when a number of cavities are clogged either those with the smallest cavity depths situated at the inlet or those with the deepest cavity depths at the outlet. It can be appreciated that any of these configurations implies degraded results, but the losses are more important when removing the cavities with the deepest depths, for both absorption and transmission coefficients, increasing the lower frequency for the ABH efficiency.

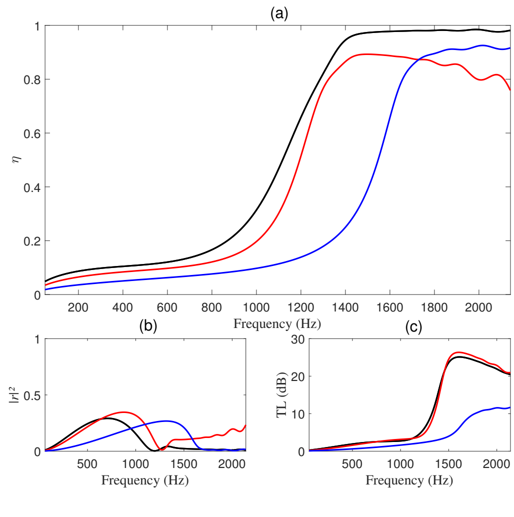

Fig. 3.The dissipation (a), reflection (b) and transmission loss (c) spectra for the nominal ABH silencer (black) and for ABHs whose 8 smallest (red) or 8 deepest (blue) cavities have been discarded.

A parameter that has been also investigated either for vibration or acoustic black holes studies is the rate of variation, m, of the ABH profile, that has here been increased progressively from $m = 0.5$ up to

$m=10$. The profiles of the cavity depths are presented in Fig. 4 (c), whereas the absorption, reflection and transmission power spectra can be seen in Figs. 4 (a, b, c) respectively.

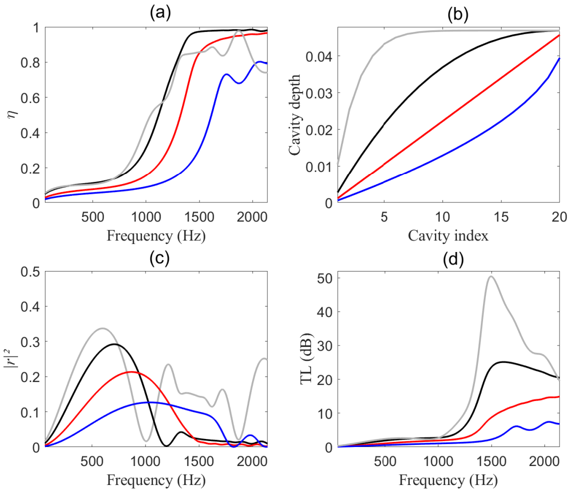

Fig. 4.The dissipation (a), reflection (c) and transmission loss (d) power spectra for ABH silencers whose rate of increase m of the cavity depths has been varied [$m=0.5$ (blue); $m=1$ (red); $m=2.4$ (black); $m=10$, grey); (b) shows the corresponding profiles of cavity depths.

The results obtained show that, as far as the sound transmission is concerned, the performance increases as the profile order also increases, achieving the best configuration for the highest order $m=10$. However, this trend does not seem to be the most advantageous when considering the dissipation and reflection coefficients, since the order $m=10$ achieves lower dissipation and larger reflection values than the one with $m=2.4$, although it still performs over a broad frequency band above 1400 Hz. Combining both the dissipation and reflection values, the profile with $m=2.4$ presents the best performance although the highest order $m=10$ is able to provide better transmission values above 1400 Hz.

4. Numerical and experimental validation

A numerical and experimental validation has been performed for confirming the analytical results. FEM computation with Comsol Viscothermal Multiphysics has been performed in the frequency domain for comparison with the TMM. Comparison between analytical and numerical results is presented in Fig. 5 with a reasonable agreement for the dissipated values over the whole frequency range.

An ABH with cavity depths profile, $m=2.4$, has been 3D printed as a single part using fused deposition modelling. The inner geometry is presented in Fig. 5 (b). It has been mounted in a standing wave tube test rig for determination of the absorption, transmission and dissipation coefficients. The measurement procedure uses two sources located at the inlet and outlet sections of the test facility, with a total length of 1000 mm, and an inner diameter of 100 mm. Four microphones are used to obtain an estimation of the scattering matrix [21] up to the first cut-on frequency at 2.1 kHz. The acquisition has been made using the OROS (OR38) multichannel system that generates a white noise at a sampling rate of 12.8 kHz and with a spectral resolution of 1.56 Hz.

The comparison between the experimental and the predicted results for dissipation, reflection and transmission can be seen in Fig. 5 (a, c, d). It can be appreciated that correlation between both estimations is good in general terms, especially for the dissipated values.

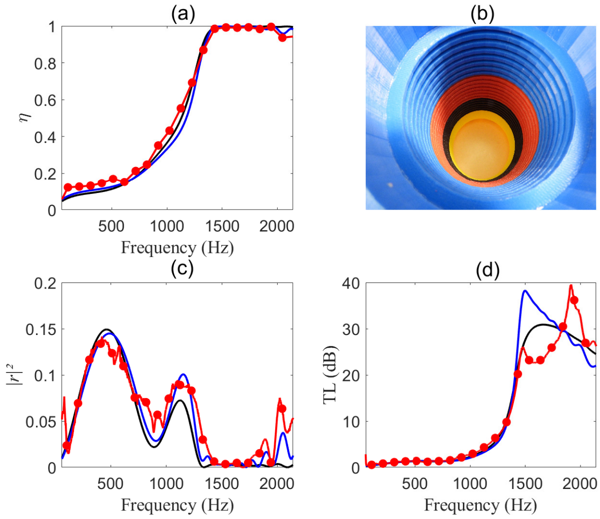

Fig. 5. Comparison between the simulated (TMM; black; FEM, blue) and measured (red) acoustical performance of the optimized ABH: dissipation (a), reflection (c) and transmission loss (d); (b) shows a picture of the 3D-printed ABH interior.

5. Conclusions

The concept of slow-sound for the design and characterization of an acoustic muffler with broadband performance has been studied. Unlike closed ABHs, a widely-opened silencer needs to take into account both reflection and transmission to enhance dissipation inside the control device. We have first predicted that performance using an analytical formulation based on the TMM, with JCAL model of visco-thermal losses, has been successfully compared against numerical results obtained with Visco-thermal Acoustics FEM Comsol Multiphysics. The analytical method has been used to perform cost-efficient parametric studies on the performance of the ABH silencer. A complete selection of the ABH optimal parameters is a high cost computational problem that is difficult to solve by FEM techniques and alternative methods have to be employed. In this work we have studied the behaviour of the ABH first when a number of cavities are removed from the optimal configuration, discarding those with the smallest cavity depths situated at the inlet or those with the deepest cavity depths at the outlet. For both cases it has been shown that performance worsens, but the effect is more significant when discarding the deepest cavities, with a dissipation spectrum upshifted towards the higher frequencies. We have also changed the increasing rate of the cavity depth profile and found a strong influence on the dissipation performance. The results have been compared against a set of measurements carried out in a standing wave tube facility for determining the dissipation achieved by the ABH muffler. It has been found that experiments follow reasonably well the expected results, confirming the validity of the TMM approach in subsequent optimization studies.

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